Chapter 6 Surface Areas and Volumes

Very Short Answer Type Questions

Q1. How much ice-cream can be put into a cone with base radius 3.5 cm and height 12 cm ? [CBSE-14-GDQNI3W]

Answer.

Here, radius $(r) = 3.5$ cm and height $(h) = 12$ cm

$$\therefore \quad \text{Amount of ice-cream} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12$$
$$= 154 \text{ cm}^3$$

Q2. The total surface area of a cube is 726 cm^2. Find the length of its edge. [CBSE-14-ERFKZ8H]

Answer.

$$\text{Total surface area of a cube} = 726 \text{ cm}^2$$
$$6 \times (\text{side})^2 = 726$$
$$(\text{side})^2 = 121$$
$$\text{side} = 11 \text{ cm}$$

Hence, the length of the edge of cube is 11 cm.

Q3. If a wooden box of dimensions 8 m x 7 m x 6 m is to carry boxes of dimensions 8 cm x 7 cm x 6 cm, then find the maximum number of boxes that can be carried in the wooden box. [CBSE-14-GDQNI3W]

Answer.

$$\text{Volume of wooden box} = 800 \text{ cm} \times 700 \text{ cm} \times 600 \text{ cm}$$
$$\text{Volume of box} = 8 \text{ cm} \times 7 \text{ cm} \times 6 \text{ cm}$$
$$\therefore \quad \text{Number of boxes} = \frac{\text{Volume of wooden box}}{\text{Volume of each box}}$$
$$= \frac{800 \times 700 \times 600}{8 \times 7 \times 6} = 1000000$$

Q4. Two cubes of edge 6 cm are joined to form a cuboid. Find the total surface area of the cuboid. [CBSE-15-NS72LP7]

Answer. When two cubes are joined end to end, then
Length of the cuboid = 6 + 6 = 12 cm
Breadth of the cuboid = 6 cm
Height of the cuboid = 6 cm
Total surface area of the cuboid = 2 (lb + bh + hi)
= 2(12 x6 + 6×6 + 6×12)
= 2(72 + 36 + 72) = 2(180) = 360 cm^2

Q5. Calculate the edge of the cube if its volume is 1331 cm^3. [CBSE-15-6DWMW5A]

Answer.

Volume of cube = 1331 cm^3

$(Side)^3 = 1331$

$$Side = (11 \times 11 \times 11)^{\frac{1}{3}} = 11 \text{ cm}$$

Q6. If in a cylinder, radius is doubled and height is halved, then find its curved surface area.

Answer.

Let r and h be radius and height of the cylinder, then C.S.A. $= 2\pi rh$

Now, radius is doubled and height is halved.

$\therefore$ New radius $= 2r$ and new height $= \frac{h}{2}$

$$\text{New C.S.A.} = 2\pi \times 2r \times \frac{h}{2} = 2\pi rh$$

Q7. The radii of two cylinders of the same height are in the ratio 4 :5, then find the ratio of their volumes.

Answer.

Let r_1 and r_2 be radii of two cylinder and V_1, V_2 be their volume.

Let h be height of the two cylinders, then

$$V_1 = \pi r_1^2 h \text{ and } V_2 = \pi r_2^2 h$$

$\therefore$ $$\frac{V_1}{V_2} = \frac{\pi r_1^2 h}{\pi r_2^2 h} = \frac{r_1^2}{r_2^2} = \frac{16}{25}$$

Q8. Find the area of the sheet required to make closed cylindrical vessel of height 1 m and diameter 140 cm.

Answer.

Required sheet = T.S.A. of cylinder

$$= 2\pi r(h + r) = 2 \times \frac{22}{7} \times \frac{70}{100}\left(1 + \frac{70}{100}\right)$$

$$= 2 \times 22 \times 0.1 \times 1.7 = 7.48 \text{ m}^2$$

Q9. Find the volume of cone of radius r/2 and height '2h'.

Answer.

$$\text{Volume of cone} = \frac{1}{3}\pi \times \left(\frac{r}{2}\right)^2 \times 2h$$

$$= \frac{1}{3}\pi \times \frac{r^2}{4} \times 2h = \frac{1}{6}\pi r^2 h \text{ cu. units}$$

Q10. How many balls, each of radius 2 cm can be made from a solid sphere of lead of radius 8 cm ? [CBSE-14-17DIG1U]

Answer.

$$\text{No. of balls} = \frac{\text{Volume of sphere}}{\text{Volume of each ball}} = \frac{\frac{4}{3}\pi \times 8 \times 8 \times 8}{\frac{4}{3}\pi \times 2 \times 2 \times 2} = 64$$

Q11. A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. Find the radius of the sphere.[NCERT Exemplar Problem]

Answer.

$$\text{Volume of cone} = \text{Volume of sphere}$$

$$\frac{1}{3}\pi(2.1)^2 \times 8.4 = \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad r^3 = \frac{(2.1)^2 \times 8.4}{4} = (2.1)^3$$

$$\Rightarrow \quad r = 2.1 \text{ cm}$$

$$\therefore \text{ Radius of the sphere} = 2.1 \text{ cm}$$

Q12. If the volume of a sphere is numerically equal to its surface area, then find the diameter of the sphere.

Answer.

Let r be the radius of the sphere.

and $\quad \text{Volume of a sphere} = \text{Surface area of the sphere}$

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = 4\pi r^2$$

$$\Rightarrow \quad r = 3 \text{ cm}$$

$$\therefore \quad \text{Diameter of the sphere} = 2r = 2 \times 3 = 6 \text{ cm}$$

Q13. The radius of a spherical balloon increases from 6 cm to 12 cm as air is being pumped into it. Then what will be the ratio of surface areas of the original balloon to the resulting new balloon ?

Answer.

Surface area of a spherical balloon whose radius is 6 cm

$$= 4\pi \times 6 \times 6 \text{ cm}^2$$

Surface area of a spherical balloon whose radius is 12 cm

$$= 4\pi \times 12 \times 12 \text{ cm}^2$$

$$\therefore \quad \text{Ratio of surface areas} = \frac{4\pi \times 6 \times 6}{4\pi \times 12 \times 12} = \frac{1}{4} = 1:4$$

Q14. The outer and the inner radii of a hollow sphere are 12 cm and 10 cm. Find its volume. [CBSE-14-17DIG1U]

Answer. Inner radius of hollow sphere (r) = 10 cm Outer radius of hollow sphere (R) = 12 cm

$$\text{Volume of sphere} = \frac{4}{3}\pi (R^3 - r^3)$$

$$= \frac{4}{3} \times \frac{22}{7} (12^3 - 10^3)$$

$$= \frac{4}{3} \times \frac{22}{7} (1728 - 1000)$$

$$= \frac{4}{3} \times \frac{22}{7} \times 728 = 3050.67 \text{ cm}^3$$

Q15. In a cylinder, if radius is halved and height is doubled, then find the volume with respect to original volume. [NCERT Exemplar Problem]

Answer.

$$\text{Here, } r = \frac{r}{2}, \ h = 2h$$

$$\therefore \quad \text{Volume of cylinder} = \pi\left(\frac{r}{2}\right)^2 2h = \frac{1}{2}\pi r^2 h$$

$$\text{Original volume of cylinder} = \pi r^2 h$$

Volume w.r.t. original volume of cylinder

$$= \frac{\frac{1}{2}\pi r^2 h}{\pi r^2 h} = \frac{1}{2}$$

Short Answer Questions Type-1

Q16. A spherical ball is divided into two equal halves. If the curved surface area of each half is 56.57 cm?, find the volume of the spherical ball. [use π = 3.14] [CBSE-14-GDQNI3W]

Answer.

Since curved surface of half of the spherical ball = 56.57 cm^2

$$\therefore \quad 2\pi r^2 = 56.57$$

$$\Rightarrow \quad r^2 = \frac{56.57}{2 \times 3.14} = 9$$

$$\Rightarrow \quad r = 3 \text{ cm}$$

$$\text{Now, volume of spherical ball} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times 3.14 \times 3 \times 3 \times 3$$

$$= 113.04 \text{ cm}^3$$

Q17. Find the length of the longest pole that can be put in a room of dimensions 10 m x 10 m x 5 m. [NCERT Exemplar Problem]

Answer. Here, we have a cuboid with dimensions
l= length = 10 m, b = breadth = 10 m and h = height = 5 m
Now, length of longest pole = diagonal of cuboid

$$\therefore \quad \text{Required length} = \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{100 + 100 + 25} = \sqrt{225} = 15 \text{ m}$$

Q18. Find the capacity in litres of a conical vessel having height 8 cm and slant height 10 cm. [CBSE-15-6DWMW5A]

Answer.

$$\text{Height of conical vessel } (h) = 8 \text{ cm}$$

$$\text{Slant height of conical vessel } (l) = 10 \text{ cm}$$

$$\therefore \quad r^2 + h^2 = l^2$$

$$r^2 + 8^2 = 10^2$$

$$r^2 = 100 - 64 = 36$$

$$r = 6 \text{ cm}$$

$$\text{Now, volume of conical vessel} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8 = 301.71 \text{ cm}^3 = 0.30171 \text{ litre}$$

Q19. Calculate the surface area of a hemispherical dome of a temple with radius 14 m to be whitewashed from outside. [CBSE -15-NS72LP7]

Answer.

Here, radius of hemispherical dome $(r) = 14$ m

$$\text{Surface area of dome} = 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times 14 \times 14 = 1232 \text{ m}^2$$

Hence, total surface area to be whitewashed from outside is 1232 m^2.

Q20. A school provides milk to the students daily in cylindrical glasses of diameter 7 cm. If the glass is filled with milk up to a height of 12 cm, find how many litres of milk is needed to serve 1600 students. [CBSE March 2011]

Answer.

$$\text{Diameter } d = 7 \text{ cm}$$

$$\text{Radius } r = \frac{7}{2} \text{ cm and } h = 12 \text{ cm}$$

$$\therefore \quad V = \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 = 462$$

$$\text{Total milk for 1600 students} = 462 \times 1600$$

$$= 739200 \text{ cm}^3$$

$$= \frac{739200}{1000} \text{ litres} = 739.2 \text{ litres.}$$

Q21. A rectangular piece of paper is 22 cm long and 10 cm wide. A cylinder is formed by rolling the paper along its length. Find the volume of the cylinder. [CBSE-14-ERFKZ8H]

Answer.

Since rectangular piece of paper is rolled along its length.

$\therefore \quad 2\pi r = 22$

$$r = \frac{22 \times 7}{2 \times 22} = 3.5 \text{ cm}$$

Height of cylinder $(h) = 10$ cm

$\therefore$ Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 10$$

$$= 385 \text{ cm}^3$$

Q22. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find it volume. If 1cm^3 wheat cost is Rs 10, then find total cost. [CBSE-14-GDQNI3W]

Answer.

Diameter of cone $= 10.5$ m

$\therefore$ Radius of cone $(r) = 5.25$ m

Height of cone $(h) = 3$ m

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 3$$

$$= 86.625 \text{ m}^3$$

Cost of 1m^3 of wheat $= ₹\ 10$

$\therefore$ Cost of $86.625\ \text{m}^3$ of wheat $= ₹\ 10 \times 86.625$

$= ₹\ 866.25$

Q23. A shot-put is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g/cm^3. Find the mass of the shot-put. [CBSE March 2012]

Answer.

We have, the radius of a metallic sphere $(r) = 4.9$ cm

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 4.9 \times 4.9 \times 4.9$

$= 493.005 \text{ cm}^3$

$\because$ Density of the metal used $= 7.8 \text{ g/cm}^3$

Hence, the mass of the shot-put $= 493.005 \times 7.8 = 3845.44$ g

Q24. A cylindrical vessel can hold 154 g of water. If the radius of its base is 3.5 cm, and 1 cm^3 of water weighs lg,find the depth of water. [CBSE-14-17DIG1U]

Answer.

Since 1 cm^3 of water weighs 1 g.

$\therefore$ Volume of cylindrical vessel = 154 cm^3

$$\pi r^2 h = 154$$

$$\frac{22}{7} \times 3.5 \times 3.5 \times h = 154$$

$$h = \frac{154 \times 7}{22 \times 3.5 \times 3.5}$$

$$h = 4 \text{ cm}$$

Hence, the depth of water is 4 cm.

Short Answer Questions Type-II

Q25. A wall of length 10 m is to be built across an open ground. The height of the wall is 5 m and thickness of the wall is 42 cm. If this wall is to be built with bricks of dimensions 42 cm x 12 cm x 10 cm, then how many bricks would be required ? [CBSE-14-ERFKZ8H] [CBSE-15-NS72LP7]

Answer.

Length of the wall = 10 m = 1000 cm

Thickness of the wall = 42 cm

Height of the wall = 5 m = 500 cm

$\therefore$ Volume of the wall = $1000 \times 500 \times 42$ cm^3

Volume of each brick = $42 \times 12 \times 10$ cm^3

$$\text{No. of bricks} = \frac{\text{Volume of the wall}}{\text{Volume of each brick}}$$

$$= \frac{1000 \times 500 \times 42}{42 \times 12 \times 10}$$

$$= 4166.67 \simeq 4167$$

Q26. The curved surface area of a cylinder is 176 cm^2 and its area of the base is 38.5 cm^2. Find the volume of the cylinder. (Take π = 22/7) [CBSE March 2012]

Answer.

$$\text{Area of base} = 38.5 \text{ cm}^2$$

$$\pi r^2 = 38.5 \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \frac{38.5}{7} \times 22 \text{ cm}^2 = 121 \text{ cm}^2$$

$$\Rightarrow \quad r = 11 \text{ cm}$$

$\because$ Curved surface area of a cylinder $= 176 \text{ cm}^2$

$$2\pi rh = 176$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times 11 \times h = 176$$

$$\Rightarrow \quad h = \frac{176 \times 7}{2 \times 22 \times 11} = \frac{28}{11} \text{ cm}$$

$$\therefore \quad \text{Volume} = \pi r^2 h$$

$$= 38.5 \times \frac{28}{11} \text{ cm}^3 = 98 \text{ cm}^3$$

Q27. The diameter of a roller is 42 cm and its length is 120 cm. It takes 500 complete revolutions to move once to land a playground. Find the area of the playground in m^2. [CBSE March 2012]

Answer.

We have, the diameter of a cylindrial roller $= 42$ cm

$\Rightarrow$The radius of a cylindrical roller $(r) = \frac{42}{2} = 21$ cm

Length of a cylindrical roller $(h) = 120$ cm

$$\text{Curved surface of the roller} = 2\pi rh = 2 \times \frac{22}{7} \times 21 \times 120 = 15840 \text{ cm}^2$$

$$= \frac{15840}{10000} \text{m}^2 = 1.584 \text{ m}^2$$

Area covered by the roller in one complete revolution

$$= \text{Curved surface of the roller} = 1.584 \text{ m}^2$$

$\therefore$ Area of the playground = Area covered by the roller in 500 complete revolutions

$$= 500 \times 1.584 = 792 \text{ m}^2$$

Q28. Rinku has built a cuboidal water tank in his house. The top of the water tank is covered with an iron lid. He wants to cover the inner surface of the tank including the base with tiles of size 10 cm by 8 cm. If the dimensions of the water tank are 180 cm x 120 cm x 60 cm and cost of tiles is f 480 per dozen, then find the total amount required for tiles. [CBSE March 2012]

Answer.

Total inner surface area of the water tank including the base without top

$$= 2(l + b) \times h + l \times b$$
$$= 2(180 + 120) \times 60 + 180 \times 120$$
$$= 36000 + 21600 = 57600 \text{ cm}^2$$

Area of each tile $= 10 \times 8 = 80 \text{ cm}^2$

$\therefore$ Number of tiles required $= \frac{57600}{80} = 720$

Total amount required for 720 tiles at the rate of ₹ 480 per dozen $= ₹ \frac{480}{12} \times 720$

$= ₹ 28800.$

Q29. The diameter of moon is approximately 1/4 th of the diameter of earth. What fraction of volume of earth is the volume of moon ? [CBSE March 2012]

Answer.

Since the diameter of moon $= \frac{1}{4}$ th of the diameter of earth

$\Rightarrow$ The radius of moon $= \frac{1}{4}$ th of the radius of earth

Let the radius of earth is r and the radius of the moon is $\frac{r}{4}$

Volume of the earth $(V_1) = \frac{4}{3}\pi r^3$

and Volume of the moon $(V_2) = \frac{4}{3}\pi\left(\frac{r}{4}\right)^3 = \frac{4}{3}\pi \times \frac{r^3}{64}$

Now, $\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi \times \frac{r^3}{64}} = \frac{64}{1} \Rightarrow V_1 : V_2 = 64 : 1$

Hence, the volume of earth : the volume of moon $= 64 : 1$

Q30. The curved surface area of a cylinder is 154 cm^2. The total surface area of the cylinder is three times its curved surface area. Find the volume of the cylinder. [CBSE-14-GDQNI3W]

Answer.

Since curved surface area of cylinder = 154 cm^2 [given]

Total surface area of cylinder = 3 × curved surface area

$$2\pi rh + 2\pi r^2 = 3 \times 154$$

$$154 + 2\pi r^2 = 462$$

$$2\pi r^2 = 462 - 154 = 308$$

$$r^2 = \frac{308 \times 7}{2 \times 22} = 49$$

$$r = 7 \text{ cm}$$

Now, $2\pi rh = 154$

$$2 \times \frac{22}{7} \times 7 \times h = 154$$

$$h = \frac{154}{44} = 3.5 \text{ cm}$$

$\therefore$ Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 3.5 = 539 \text{ cm}^3$$

Q31. A right angled A ABC with sides 3 cm, 4 cm and 5 cm is revolved about the fixed side of 4 cm. Find the volume of the solid generated. Also, find the total surface area of the solid. [CBSE-14-17DIG1U]

Answer.

When rt. ∠ed ΔABC is revolved about AB = 4 cm, it forms a right circular cone of radius 3 cm and height 4 cm. Slant height of the cone is 5 cm.

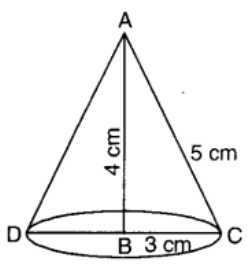
$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 4$$

$$= 37.71 \text{ cm}^3$$

Total surface area of the solid $= \pi r^2 + \pi rl = \pi r (r + l)$

$$= \frac{22}{7} \times 3 \times 8 = 75.43 \text{ cm}^2$$



Q32. Curved surface area of cylindrical reservoir 12 m deep is plastered from inside with concrete mixture at the rate of Rs 15 per m^2. If the total payment made is of Rs 5652, then find the capacity of this reservoir in litres. [CBSE-14-GDQNI3W]

Answer.

Total cost of plastering from inside of a cylindrical reservoir = ₹5652

Cost of 1 m^2 = ₹15

$\therefore$ Curved surface area = $\frac{5652}{15}$ = 376.8 m^2

Height of reservoir (h) = 12 m

$\therefore$ $2\pi rh$ = 376.8

$$2\times\frac{22}{7}\times r\times 12 = 376.8$$

$$r = \frac{376.8\times 7}{2\times 22\times 12} \simeq 5 \text{ m}$$

Volume of reservoir = $\pi r^2 h$

$$= \frac{22}{7} \times 5 \times 5 \times 12$$

$$= 942.857 \text{ m}^3$$

$$= 942.857 \times 1000 \text{ litres}$$

$$= 942857 \text{ litres}$$

Hence, the capacity of given reservoir is 942857 litres.

Q33. How many metres of 5 m wide cloth will be required to make a conical tent, the radius of whose base is 3.5 m and height is 12 m ? [CBSE March 2011]

Answer.

$$l = \sqrt{h^2+r^2} = \sqrt{(3.5)^2+(12)^2} = \sqrt{12.25+144}$$

$$= \sqrt{156.25} = 12.5 \text{ m}$$

Curved surface area = $\pi rl = \frac{22}{7} \times 3.5 \times 12.5 = 137.5 \text{ m}^2$

Area of cloth = 137.5 m^2

Length of cloth required = $\frac{\text{C.S.A.}}{\text{Width}}$

$$l = \frac{137.5}{5} = 27.5 \text{ m}$$

Q34. A shopkeeper has one spherical laddoo of radius 5 cm. With the same amount of material, how many laddoos of radius 2.5 cm can be made ? [NCERT Exemplar Problem]

Answer.

Radius of a big spherical laddoo = 5 cm

$\therefore$ Volume of the big spherical laddoo = $\frac{4}{3}.\pi \times 5 \times 5 \times 5\ cm^3$

Radius of a small spherical laddoo = $2.5 = \frac{5}{2}$ cm

$\therefore$ Volume of the small spherical laddoo = $\frac{4}{3}\pi \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}\ cm^3$

Let n be the number of small laddoos that can be made using the same amount (volume) of big laddoo.

Therefore, $n \times \frac{4}{3}\pi \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{4}{3}\pi \times 5 \times 5 \times 5$

$\Rightarrow$ $n = 2 \times 2 \times 2 = 8$

Hence, with the same amount of big laddoo, 8 small laddoos can be made.

Q35. A semicircular sheet of metal of radius 14 cm is bent to form an open conical cup. Find the capacity of the cup.

Answer.

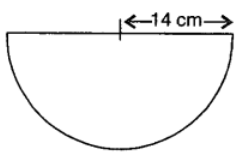
Radius of semicircular sheet (r) = 14 cm

$\therefore$ slant height (l) = 14 cm

Circumference of base = Circumference of semicircular sheet

$2\pi r = \pi \times 14$

$r = 7$ cm



$\therefore$ $r^2 + h^2 = l^2$

$\Rightarrow$ $h^2 = l^2 - r^2 = 14^2 - 7^2 = 196 - 49 = 147$

$\Rightarrow$ $h = \sqrt{147} = 12.12$ cm

Now, volume of conical cup = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12.12 = 622.16\ cm^3$

Long Answer Type Questions

Q36. A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceed its height ?[NCERT Exemplar Problem]

Answer. Let 'r' be the radius of sphere and right circular cylinder with height 'h'. According to the statement of question, we have

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$

$\Rightarrow$ $\frac{4}{3}r = h \Rightarrow r = \frac{3}{4}h$

Since radius = $\frac{3}{4} \times$ height of cylinder

Therefore, diameter of cylinder = $2 \times r = \frac{2 \times 3}{4}h = \frac{3}{2}h$

Total increase in height = $\left(\frac{3}{2} - 1\right)h = \frac{1}{2}h$

$\therefore$ Required percentage = $\frac{\frac{1}{2}h}{h} \times 100 = 50$

Q37. A cube and a cuboid have the same volume. The dimensions of the cuboid are in the ratio 1:2: 4. If the difference between the cost of painting the cuboid and cube (whole surface area) at the rate of Rs 5 per

m^2 is Rs 80. Find their volumes. [CBSE March 2012]

Answer.

Let the dimensions of the cuboid are x, $2x$ and $4x$ respectively.

$\therefore$ Volume of the cuboid $= x \times 2x \times 4x = 8x^3$

Since Volume of the cube = Volume of the cuboid

$\Rightarrow$ Volume of the cube $= 8x^3$

$\Rightarrow$ Edge of the cube $= \sqrt[3]{8x^3} = 2x$

Now, Surface area of the cuboid $= 2(x \times 2x + 2x \times 4x + 4x \times x)$

$= 2(2x^2 + 8x^2 + 4x^2) = 2 \times 14x^2 = 28x^2$

Surface area of the cube $= 6 \times (2x)^2 = 6 \times 4x^2 = 24x^2$

Now, cost of painting the cuboid at the rate of ₹ 5 per $m^2 = 5 \times 28x^2 =$ ₹ $140\,x^2$

and cost of painting the cube at the rate of ₹ 5 per $m^2 = 5 \times 24x^2 =$ ₹ $120\,x^2$

Difference between these costs = ₹ 80

$\Rightarrow$ $140x^2 - 120x^2 = 80$

$\Rightarrow$ $20x^2 = 80$

$\Rightarrow$ $x^2 = \frac{80}{20} = 4$

$\Rightarrow$ $x = 2$ m

Hence, volume of each of cuboid and cube $= 8x^3 = 8 \times 2^3 = 8 \times 8 = 64\ m^3$.

Q38. Ajay has built a cubical witter tank in his house. The top of the water tank is covered with lid. He wants to cover the inner surface of the tank including the lid with square tiles of side 25 cm. If each inner edge of the water tank is 2 m long and tiles costs Rs 360 per dozen, then find the total amount required for tiles. [CBSE March 2012]

Answer.

We have, Edge of the cube$(a) = 2$ m $= 200$ cm

$\therefore$ Area of remaining six faces of tank $= 6a^2 = 6 \times (200)^2$

$= 6 \times 40000 = 240000\ cm^2$

Area of the square tile $= 25 \times 25 = 625\ cm^2$

$$\text{Number of square tiles required} = \frac{\text{Area of six faces of tank}}{\text{Area of a square tile}} = \frac{240000}{625} = 384$$

Cost of one dozen tiles = ₹ 360

$$\text{Cost of one tile} = \frac{360}{12} = ₹\ 30$$

$\therefore$ Total amount paid on 384 tiles $= 384 \times 30 =$ ₹ 11520.

Q39. A tent is in shape of a right circular cylinder up to a height of 3 m and a cone above it. The maximum height of the tent above ground is 13.5 m. Calculate the cost of painting the inner side of the tent at the rate of Rs 3 per sq. m, if the radius of the base is 14 m. [CBSE March 2011]

Answer.

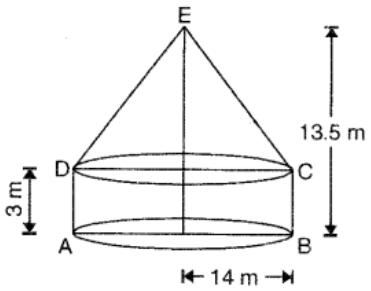
Curved surface area of cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 3 = 264 \text{ m}^2$$

Height of cone $= 13.5 - 3 = 10.5$ cm

Now, $l = \sqrt{r^2 + h^2} = \sqrt{14^2 + 10.5^2}$

$$= \sqrt{196 + 110.25} = \sqrt{306.25} = 17.5 \text{ m}$$



Curved surface area of cone $= \pi rl = \frac{22}{7} \times 14 \times 17.5 = 770 \text{ m}^2$

Total inner curved surface area $= 770 + 264 = 1034 \text{ m}^2$

Cost per $\text{m}^2 = ₹ 3$

$\Rightarrow$ Total cost $= 1034 \times 3 = ₹ 3102$

Q40. Manoj Sweets placed an order of making 30 cm x 20 cm x 6 cm cardboard boxes for packing their sweets. For all overlaps, 5 % of total area is required extra. If cost of the cardboard is Rs 2 for 1000 cm^2, find the cost of the cardboard used for making 500 boxes. [CBSE-14-17DIG1U]

Answer.

Total surface area of a box $= 2\,(30 \times 20 + 20 \times 6 + 6 \times 30)$

$= 2\,(600 + 120 + 180)$

$= 2\,(900) = 1800 \text{ cm}^2$

Total surface area of 500 boxes $= 500 \times 1800$

$= 900000 \text{ cm}^2$

Surface area for overlaps $= \frac{5}{100} \times 900000$

$= 45000 \text{ cm}^2$

Total surface area of cardboard $= 900000 + 45000$

$= 945000 \text{ cm}^2$

Cost of cardboard is ₹ 2 for 1000 cm^2

$\therefore$ Total cost of the cardboard $= ₹ \frac{2}{1000} \times 945000$

$= ₹ 1890$

Q41. A cylindrical bucket 32 cm high and with base diameter 36 cm is filled with wheat. This bucket is emptied on the ground and a conical heap is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.[CBSE-14-ERFKZ8H]

Answer.

Height of the cylindrical bucket = 32 cm

Radius of the cylindrical bucket = 18 cm

$\therefore$ Volume of the bucket = $\pi r^2 h$

$= \pi \times 18 \times 18 \times 32 \text{ cm}^3$

Height of the conical heap (H) = 24 cm

Let radius of the conical heap be R cm

$\therefore$ Volume of conical heap = Volume of cylindrical bucket

$$\frac{1}{3}\pi R^2 H = \pi \times 18 \times 18 \times 32$$

$$R^2 = \frac{18 \times 18 \times 32 \times 3}{24} = 1296$$

$$R = 36 \text{ cm}$$

$$\text{Slant height } (l) = \sqrt{R^2+H^2} = \sqrt{36^2+24^2}$$

$$= \sqrt{1296+576} = \sqrt{1872} = 43.27 \text{ cm}$$

Q42. Using clay, Anant made a right circular cone of height 48 cm and base radius 12 cm. Versha reshapes it in the form of a sphere. Find the radius and curved sutface area of the sphere so formed. [CBSE-15-6DWMW5A]

Answer.

Height of cone (h) = 48 cm

Radius of the base of cone = 12 cm

Let R be the radius of sphere so formed

$\therefore$ Volume of sphere = Volume of cone

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

$$4R^3 = 12 \times 12 \times 48$$

$$R^3 = 12 \times 12 \times 12$$

$$R = 12 \text{ cm}$$

Now, curved surface area of sphere = $4\pi R^2$

$$= 4 \times \frac{22}{7} \times 12 \times 12$$

$$= 1810.29 \text{ cm}^2$$

Q43. A metallic right circular cylinder is 15 cm high and the diameter of its base is 14 cm. It is melted and recasted into another cylinder with radius 4 cm. Find its height and curved surface area of the new cylinder. [CBSE-14-17DIG1U]

Answer.

Radius of right circular cylinder (r) = 7 cm

Height of right circular cylinder (h) = 15 cm

$$\text{Volume of right circular cylinder} = \frac{22}{7} \times 7 \times 7 \times 15 = 2310 \text{ cm}^3$$

Radius of new cylinder (R) = 4 cm

Let height of new cylinder be H cm.

Volume of new cylinder = Volume of old cylinder

$$\frac{22}{7} \times 4 \times 4 \times H = 2310$$

$$\Rightarrow \quad H = \frac{2310 \times 7}{22 \times 4 \times 4} = \frac{735}{16} = 45.94 \text{ cm}$$

Curved surface area of new cylinder = $2\pi RH$

$$= 2 \times \frac{22}{7} \times 4 \times \frac{735}{16} = 1155 \text{ cm}^2$$

Q44. A spherical metallic shell with 10 cm external diameter weighs 1789 1/2 g. Find the thickness of the shell, if the density of the metal is 7g/cm^3. [CBSE-15-6DWMW5A], [CBSE-15-NS72LP7]

Answer. External radius of metallic shell (R) = 5 cm

Let internal radius of metallic shell be r.

$$\text{Weight of spherical metallic shell} = 1789\frac{1}{3} \text{ g} = \frac{5368}{3} \text{ g}$$

Density of the metal is 7 g/cm^3

$$\therefore \quad \text{Volume of spherical shell} = \frac{5368}{3 \times 7} \text{ cm}^3$$

$$\frac{4}{3}\pi (R^3 - r^3) = \frac{5368}{3 \times 7}$$

$$\Rightarrow \quad \frac{4}{3} \times \frac{22}{7}(5^3 - r^3) = \frac{5368}{3 \times 7}$$

$$\Rightarrow \quad 125 - r^3 = \frac{5368}{3 \times 7} \times \frac{3 \times 7}{4 \times 22}$$

$$\Rightarrow \quad 125 - r^3 = 61$$

$$\Rightarrow \quad r^3 = 125 - 61 = 64$$

$$\Rightarrow \quad r = 4 \text{ cm}$$

Hence, thickness of spherical metallic shell is (5 – 4)cm *i.e.*, 1 cm.

Q45. A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of Rs 498.96. If the rate of whitewashing is Rs 4 per square metre, find the :
(i) Inside surface area of the dome
(ii) Volume of the air inside the dome [CBSE-15-6DWMW5A]

Answer. Here, dome of building is a hemisphere.

Total cost of white washing inside the dome = Rs 498.96

Rate of whitewashing = Rs 4 per m^2

$$\therefore \quad \text{Inside surface area of the dome} = \frac{498.96}{4} = 124.74 \text{ m}^2$$

$$\therefore \quad 2\pi r^2 = 124.74$$

$$2 \times \frac{22}{7} \times r^2 = 124.74$$

$$r^2 = \frac{124.74 \times 7}{2 \times 22}$$

$$\Rightarrow \quad r^2 = 19.845$$

$$\Rightarrow \quad r = 4.45 \text{ cm}$$

$$\text{Volume of the air inside the dome} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 4.45 \times 4.45 \times 4.45$$

$$= 184.63 \text{ cm}^3$$

Value Based Questions

Q1. A residential house society is built is 4000 sq. m area. It has an underground tank to collect the rain water, the length, breadth and height of which are 50 m, 40 m and 4 m respectively. If it rains at the rate of 2 mm per minute for 5 hours, then calculate the depth of water in the tank. What value is depicted in this problem ? [CBSE-15-6DWMW5A]

Answer.

Built area of housing society = 4000 m^2

Rate of rain per minute = 2 mm

Rate of rain per hour = 2 × 60 mm

$$\text{Total rain for 5 hours} = 2 \times 5 \times 60 \text{ mm} = \frac{2 \times 5 \times 60}{10 \times 100} \text{ m}$$

$$= 0.6 \text{ m}$$

Total volume of water collected = 4000 × 0.6 m^3 = 2400 m^3

Let h be the depth of water in the tank

$$\therefore \quad 50 \times 40 \times h = 2400$$

$$h = \frac{2400}{50 \times 40} = 1.2 \text{ m}$$

Thus, depth of water in the tank is 1.2 m

Save water, save earth.

Q2. A village having a population of 4000 requires 150 litres of water per head per day. Due to lack of sources of water, they collect the water into a tank measuring 20 m x 15 m x 6 m from a river using a long pipe.
(i) For how many days will the water of this tank last ?
(ii) Which message is conveyed by the people of village ?

Answer.

(*i*) Here, the population of the village = 4000

Requirement of water per head per day = 150 litres

∴ Total requirement of water per day = 4000 × 150 litres = 600000 litres

Volume of water tank = 20 × 15 × 6 = 1800 m^3

= 1800 × 1000 litres

Now, number of days for which water of the tank will last

$$= \frac{1800 \times 1000}{600000} = 3 \text{ days}$$

Hence, water tank can serve for 3 days.

(*ii*) Save water.

Q3. Arihant builds a room measuring roof 22 m by 20 m. He also builds a cylindrical tank having diameter of base 2 m and height 3.5 m adjoining the room to collect the rainwater of roof for harvesting. If the tank is just filled with rainwater, find the rainfall in cm. What values are depicted in Arihant's plan ?

Answer.

(*i*) We have, radius of cylindrical tank (r) = 1 m

and height of cylindrical tank (h) = 3.5 m

volume of cylindrical tank = $\pi r^2 h$

$$= \frac{22}{7} \times 1 \times 1 \times 3.5 = 11 \text{ m}^3$$

Let the rainfall be h m, then

volume of water on the roof = volume of cylindrical tank

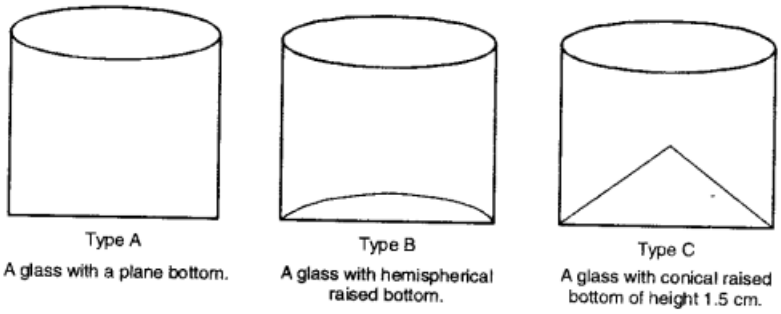
$\Rightarrow$ $22 \times 20 \times h = 11$

$\Rightarrow$ $$h = \frac{11}{22 \times 20} = \frac{1}{40} \text{ m} = \frac{100}{40} \text{ cm}$$

$$= 2.5 \text{ cm}$$

(*ii*) Save water to save earth.

Q4. Naresh, a juice seller has set up his juice shop. He has three types of glasses (see figure) of inner diameter 5 cm to serve the customers. The height of the glasses is 10 cm.



Type A
A glass with a plane bottom.

Type B
A glass with hemispherical raised bottom.

Type C
A glass with conical raised bottom of height 1.5 cm.

He decided to serve the customer in 'A' type of glasses. (Take π = 3.14)
(i) Find the volume of each type of glass.
(ii) Which glass has the minimum capacity ?
(iii) Which mathematical concept is used in above problem ?
(iv) By choosing a glass of type A, which value is depicted by juice seller Naresh?

Answer.

(*i*) Volume of glass A $= \pi r^2 h$

$= 3.14 \times 2.5 \times 2.5 \times 10 = 196.25 \text{ cm}^3$

Volume of hemisphere in glass B $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5$

$= 32.71 \text{ cm}^3$

$\therefore$ Volume of glass B = Volume of glass A – Volume of hemisphere

$= 196.25 - 32.71 = 163.54 \text{ cm}^3$

Now, Volume of cone of glass C $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 2.5 \times 2.5 \times 1.5$

$= 9.81 \text{ cm}^3$

Volume of glass C $= 196.25 - 9.81 = 186.44 \text{ cm}^3$

(*ii*) The glass of type B has minimum capacity of 163.54 cm^3

(*iii*) Volume of solid figures

(*iv*) Honesty